A stochastic approach of thermal fatigue crack growth in mixing tee piping systems from NPP

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Outline

- Introduction
- Background on stochastic models for FCG
- Basic mathematical tools for stochastic fatigue analysis
- The frequency response function method
- Development of stochastic model for thermal FCG:
  - Statistical properties of thermal spectrum;
  - Modeling of the stress response to stochastic thermal input;
  - The SIF frequency response function;
  - PSD of SIF and its spectral moments;
  - Expected value of thermal fatigue crack growth;
  - Lifetime assessment for thermal fatigue crack growth in mixing tees
- Application
- Conclusions

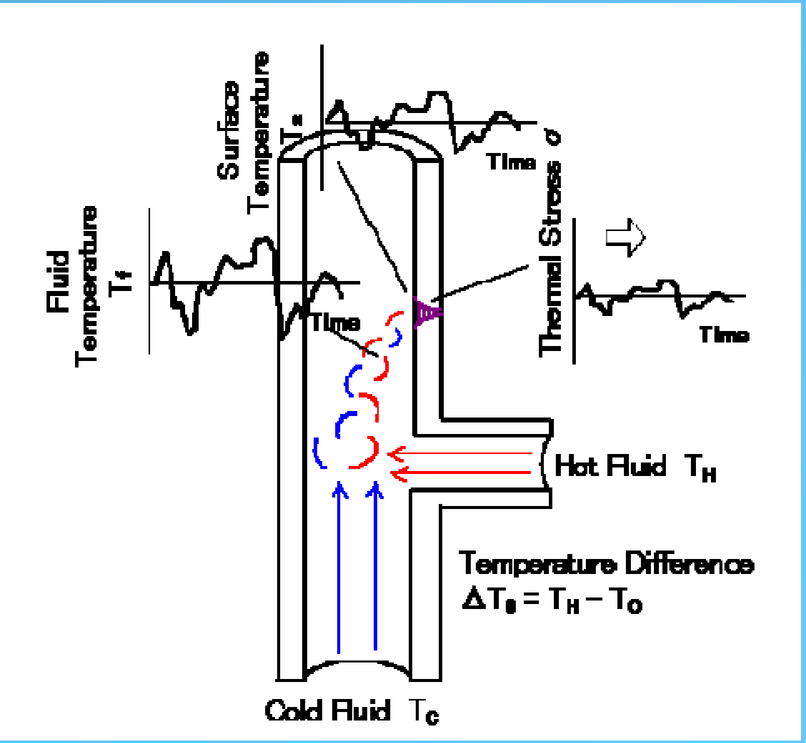
Introduction (1)

- The problem of thermal fatigue in mixing areas arises in pipes where a turbulent mixing or vortices produce rapid fluid temperature fluctuations with random frequencies.
- Structures exposed to such temperature fluctuations can suffer thermal fatigue damage and, subsequently, cracking phenomena, which can produce through wall cracks.
- Thermal striping is defined as a random temperature fluctuation produced by incomplete mixing of fluid streams at different temperatures.
- It can arise in certain light water reactor, but also in certain fast breeder reactor structures.
- Thermal striping remains a timely subject in the structural integrity area, also for future fast spectrum reactors [i], mainly with the objective of establishing thermal striping limits or appropriate screening criteria.

- [i] P. Chellapandi, S.C. Chetal, Baldev Raj, Thermal striping limits for components of sodium cooled fast spectrum reactors, Nuclear Engineering and Design 239 (2009), pp. 2754-2765

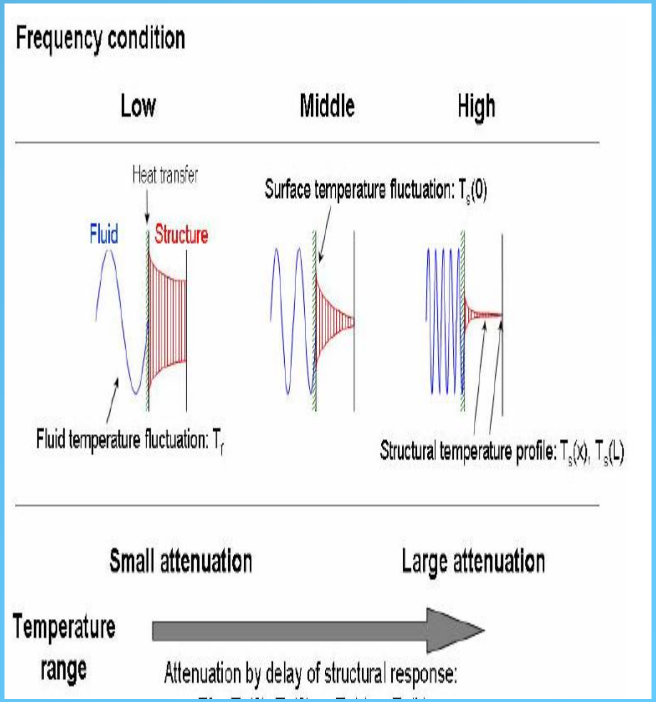
Introduction (2)

- The results in temperature fluctuations can be local or global and induce random variations of local temperature gradients in structural walls of pipe, which lead to cyclic thermal stresses.
- These cyclic thermal stresses are caused by oscillations of fluid temperature and the strain variations result in fatigue damage, cracking and crack growth.
- In particular, accurate representation of the load is a complex issues and much effort continues to be devoted to experimental and analytical studies in the area.



Introduction (3)

- The determination of the influence of such a random process on subsurface temperatures is of great importance in establishing the proper depth at which temperature sensitive becomes a concern
- The effect of spatial incoherence in surface temperature fluctuation can be used to calculate the mean square stresses and the mean square equivalent strain range, that may be used as a measure of crack initiation likelihood [i].
- By assuming a perfect spatially coherence but a temporal incoherence it was developed a method of calculating the crack propagation using linear elastic fracture mechanics and stochastic properties of temperature spectrum [ii]



- [i] A.G. Miller, Equivalent strain range due to random thermal fluctuations: effect of spatial incoherence, International Journal of Pressure vessels and Piping, 8 (1980), pp.105-130
- [ii] A.G. Miller Crack propagation due to random thermal fluctuation: effect of temporal incoherence, International Journal of Pressure vessels and Piping, 8 (1980), pp.15-24].

Introduction (4)

- Methodology steps:
  - Derive the frequency response function (FRF) for temperature through wall thickness;
  - Derive FRF for thermal hoop stress in the pipe wall
  - Derive the magnitude FRF for SIF due to thermal stress
  - Modeling the PSD of temperature spectrum
  - Derive PSD of SIF and corresponding spectral moments
  - Derive the expected value of thermal fatigue crack growth rate
  - Establish the limit state function for PFM input
  - Coupling of stochastic model with probabilistic input in Paris law to apply MCS and assessment of Pf

Background on stochastic models (1)

- The parameters that affect structural fatigue performance may include applied stress (or loading, in general), geometry of component and structural characteristics, material properties and operating environment agents. Paris law: $$\frac{da}{dN} = C \cdot \left(\Delta K \right)^n$$
- Investigation of the randomness of fatigue crack growth rate under service loading conditions should consider the randomness of loadings that gives rise to fatigue under variable amplitude loads and also the statistical characteristics of the crack growth law under constant amplitude loadings
- Several probabilistic models for crack growth are reviewed in [i], most of them suggested to "randomize" the deterministic crack propagation by a stochastic process *X(t)*: $$\frac{da(t)}{dt} = X(t) F\left(a, \Delta K, K_{\max}, R, S\right)$$

- [i]J.T.P. Yao, F. Kozin, et al. Stochastic fatigue, fracture and damage analysis, Structural Safety, 3 (1986) pp.231-267

Background on stochastic models (2)

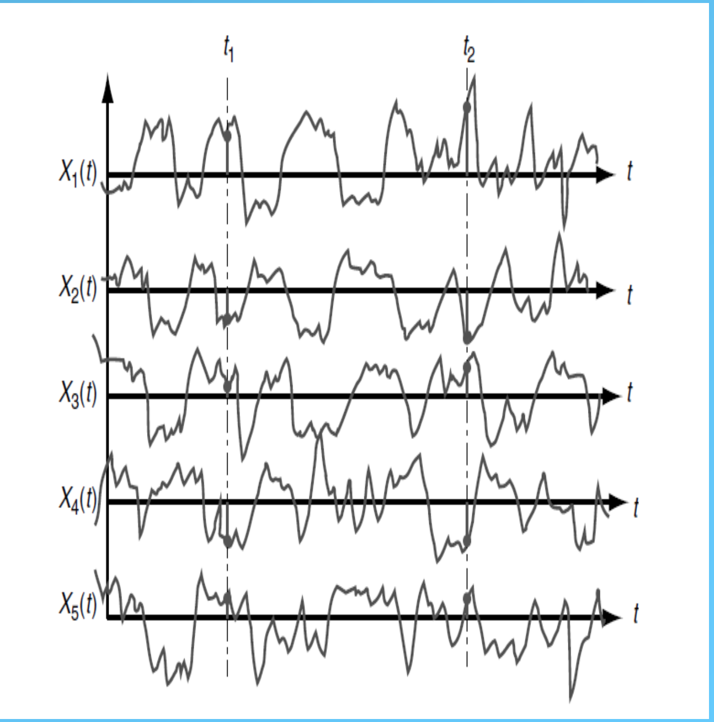
- Few stochastic models of cumulative fatigue damage process:
  - **Random Variable model.** This model regards parameters in the crack growth law simply as random variables ;
  - **Markov Chain Model.** Bogdanoff and Kozin first proposed this model in which both state and time are handled discretely with a lot of practical applications;
  - **Stochastic Differential Equation Model**. This is a such model as to treat the crack growth law as a stochastic differential equation (Fokker-Plank) by introducing temporal fluctuations into parameters in the law;
  - **Locally Averaged Model.** The model is to discuss statistical properties of whole life by summing up locally averaged lives in order to realize the constraint condition of the constant correlation distance;
  - **Renewal Process Model**. This model represents the state transition in terms of non-homogeneous Poisson processes with application of the stochastic point process,.

Basic mathematical tools for stochastic fatigue analysis (1)

- Fatigue analysis is often performed in the time domain, in which all input loading and output stress or strain response are time-based signals.
- In some situations, however, the response stress and input loading are preferable expressed as frequency-based signals, usually in the form of a power spectral density (PSD) plot.
- For this case a system function (or a characteristic of the structural system) is required to relate an input PSD of loading to the output PSD of response.
- Generally speaking, the PSD represents the energy of the time signal at different frequencies, and it is another way of denoting the loading signal in time domain.
- The transform of loading history between the time domain and frequency domain is subject to certain requirements, as per which the signal must be stationary, random, and Gaussian (normal).

Basic mathematical tools for stochastic fatigue analysis (2)

- Thermal striping is a random phenomenon in a temporal sense
- We use some mathematical tools from stochastic processes theory to develop the stochastic approach of thermal fatigue crack growth in high cycle domain. [i].
  A random process (or sometime called the stochastic process) may be thought as a "sample" function of time such as $X_1(t)$.
- A collection of an infinite number of sample time histories, such as $X_1(t)$, $X_2(t)$, $X_3(t)$, etc., makes up the random process $X(t)$.



[i] A. Papoulis, Probability, Random Variables, and Stochastic Processes, McGraw-Hill, New York , 1984

Basic mathematical tools for stochastic fatigue analysis (3)

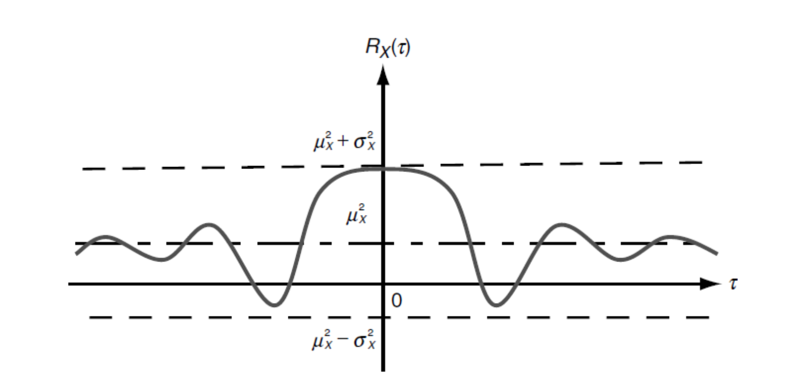
- The autocorrelation function (the process is called weakly stationary):

$$R(\tau) = E\left[X(t_1)\cdot X(t_1+\tau)\right]$$

- When τ*=0* the autocorrelation function is

$$R_X(\tau=0) = E[X(t_1)\cdot X(t_2)] = \sigma_X^2 + \mu_X^2 = E\left[X^2\right]$$

  - and *R(0)* becomes the mean square value of the process when *μX=0*.
- In Figure is shown the properties of autocorrelation function *R(*τ*)* of a stationary process *X(t).* When time interval τ goes to infinity, the random variables at *t1* and *t2* are not correlated.



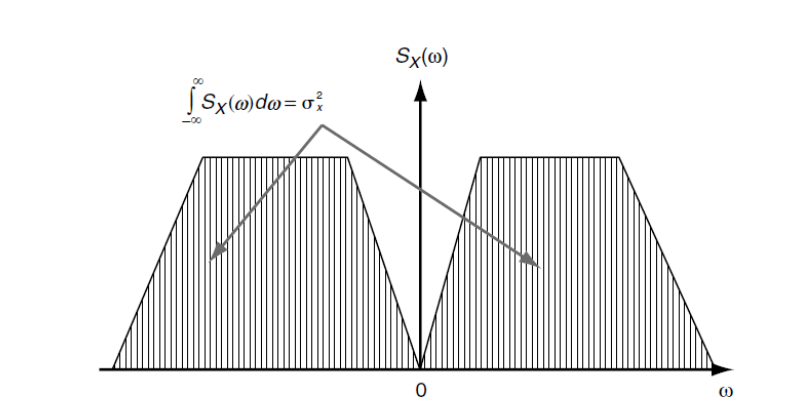
Basic mathematical tools for stochastic fatigue analysis (4)

- If the stationary random process *X(t)* is adjusted (or normalized) to a zero-mean value

$$S_X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_X(\tau) e^{-i\omega\tau} d\tau \qquad R_X(\tau) = \int_{-\infty}^{\infty} S_X(\omega) e^{i\omega\tau} d\omega$$

  - With τ=0 the variance has the form

$$E\left[X^2\right] = R_X(0) = \int_{-\infty}^{\infty} S_X(\omega) d\omega = \sigma_X^2$$

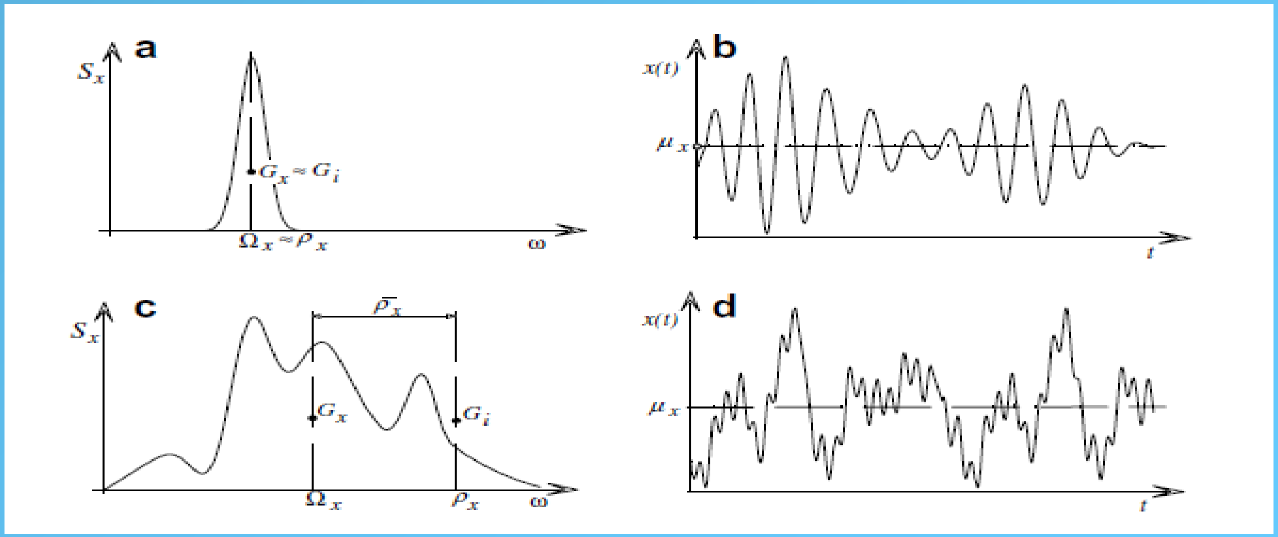


Basic mathematical tools for stochastic fatigue analysis (5)

- The two-sided spectral density, *Sx(ω),* can be transformed into an equivalent one-sided spectral density, *Wx(f)* as follows:

$$E\left[X^2\right] = \sigma_X^2 = \int_0^\infty W_X(f)df \qquad W_X(f) = 4\pi S_X(\omega)$$

- A stationary random process is called a narrow-band process if its PSD has relevant values into a limited frequency interval (narrow band of frequencies) (a); a Gaussian stationary narrow-band process is smooth and harmonic (b); a process is wide-band if its PSD has significant values into an interval of frequency(c); a wide-band process is more irregular (d)



Basic mathematical tools for stochastic fatigue analysis (6)

- The expected rate of zero up-crossing and the expected rate of peak crossing:

$$\dot{N}_{X,0} = E[\upsilon_0^+] = \sqrt{\frac{\int_0^\infty f^2 W_X(f)df}{\int_0^\infty W_X(f)df}} \qquad \dot{N}_{X,p} = E[\upsilon_p] = \sqrt{\frac{\int_0^\infty f^4 W_X(f)df}{\int_0^\infty f^2 W_X(f)df}}$$

- The distribution of amplitudes, *A*, for a stationary narrow-band Gaussian process, is characterized by the Rayleigh distribution. The probability density function for amplitudes

$$f_A(a_A) = \frac{a_A}{\sigma_A^2} \exp\left[-\frac{1}{2}\left(\frac{a_A}{\sigma_A^2}\right)\right]$$

- The *mth* moments of Rayleigh are given by

$$\mu_m = \int_0^\infty a_A^m \cdot f_A(a_A)\, da_A = \sigma_A^m 2^{\frac{m}{2}} \Gamma\left(1 + \frac{1}{m}\right)$$

Basic mathematical tools for stochastic fatigue analysis (7)

- To generate a Gaussian process, *X(t),* from a power spectral density. There are two widely used approaches, involving deterministic spectral amplitudes (DSA) and random spectral amplitudes (RSA). RSA:

$$X(t_i) = \sqrt{2}\sum_{k=0}^{N-1} A_k \cos(\omega_k t_i + \phi_k) \qquad A_k = \sqrt{R_k S(\omega_k)\Delta\omega} \qquad \omega_k = k\Delta\omega = k\frac{\omega_u}{N}$$

- ω_u is the cut-off frequency, *Rk* are of Rayleigh distribution ϕk are uniform variation within U[0,2π]

- A realization from RSA is more accurate reflection of the irregularity of real process, especially when the *mth* power of amplitudes of a process is major interest. Moreover, the process generated by RSA method is always Gaussian.

The frequency response method (1)

- Let consider *X(t)* and *Y(t)* the input and output respectively, the Fourier transform are

$$X(\omega)=\int_{-\infty}^{\infty} X(t)e^{-i\omega t}dt \qquad Y(\omega)=\int_{-\infty}^{\infty} Y(t)e^{-i\omega t}dt$$

$$Y(\omega)=H(\omega)X(\omega)$$

- The frequency response function $H(\omega)$ is useful in relating input and output power spectral densities, respectively $S_X(\omega)$ with $S_Y(\omega)$

$$S_Y(\omega)=H^*(\omega)\cdot H(\omega)\cdot S_X(\omega)=\left|H(\omega)\right|^2 S_X(\omega)$$

$$\sigma_Y^2=\int_{-\infty}^{\infty} S_Y(\omega)d\omega=\int_{-\infty}^{\infty}\left|H(\omega)\right|^2 S_X(\omega)d\omega$$

The frequency response method (2)

- Let consider the steady-state response of a linear single-input, single-output system (SISO) [i] to a real sinusoidal input of the form

$$X(t) = A\sin(\omega t)$$

- Then the sinusoidal response is

$$Y(t) = A\left|H(\omega)\right|\sin\left(\omega t + \phi(\omega)\right)$$

- The steady state response of a linear single-input, single-output system to a sinusoidal input $X(t)=A\sin\omega t$ can be characterized in terms of
  - the magnitude of the frequency response function $\left|H(\omega)\right|$
  - and the phase shift $\phi(\omega)=\angle H(\omega)$.

- [i] Saeed V. Vaseghi, Advanced digital signal processing and noise reduction, John Wiley & Sons Ltd, 2006

Development of stochastic model for thermal FCG(1)
-Statistical properties of thermal spectrum

- The 1 D heat diffusion equation in cylindrical coordinates and with axi-symmetric thermal variations is

$$\frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r}\frac{\partial \Theta}{\partial r} = \frac{1}{k}\frac{\partial \Theta}{\partial t}$$

r - radial distance;

k - the thermal diffusivity which is defined as:

$$k = \frac{\lambda}{\rho c}$$

λ – the thermal conductivity;

ρ - the mass density;

c – the specific heat coefficient.

Development of stochastic model for thermal FCG(2) -Statistical properties of thermal spectrum

- Let consider applied at the inner surface of cylinder

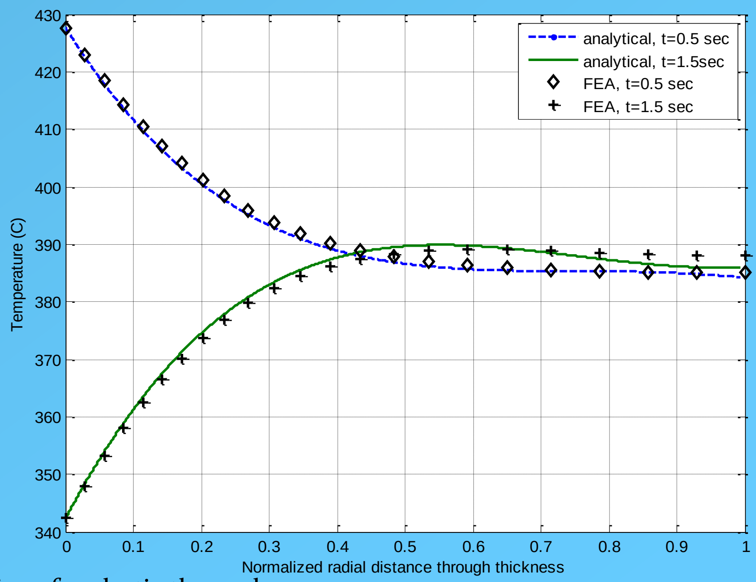
$$q(t) = \Theta_0 \cdot \sin(\omega t) = \Theta_0 \cdot \sin(2\pi f t)$$

- The analytical solution for temperature distribution through wall-thickness has been developed in a previous paper [i].

$$\Theta(r,\omega,t) = k\pi \sum_{n=1}^{\infty} \frac{s_n^2 J_0^2(s_n r_o)}{J_0^2(s_n r_o) - J_0^2(s_n r_i)} \left[J_0(s_n r) Y_0(s_n r_i) - J_0(s_n r_i) Y_0(s_n r) \right] \times$$

$$\times \left[\Theta_0 \frac{\omega e^{-k s_n^2 t} + (k s_n^2) \sin(\omega t) - \omega \sin(\omega t)}{(k s_n^2)^2 + \omega^2} \right]$$

$$J_0(s_n r_o) Y_0(s_n r_i) - J_0(s_n r_i) Y_0(s_n r_o) = 0$$



[i] V. Radu, N. Taylor, E. Paffumi, Development of new analytical solutions for elastic thermal stress components in a hollow cylinder under sinusoidal transient thermal loading, International Journal of Pressure Vessels and Piping, 2008, 85, 885-893

Development of stochastic model for thermal FCG(3)

-Statistical properties of thermal spectrum

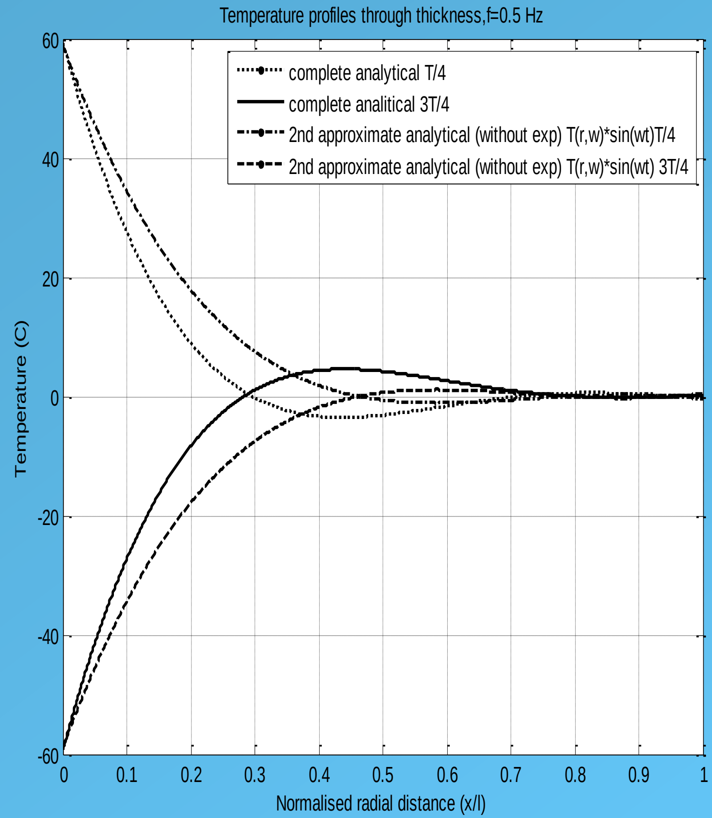
- During this study the magnitude of temperature frequency response is developed as

$$\left|H_T(r,\omega)\right| = \sum_{n=1}^{\infty} abs\left\{ \frac{k\pi \dfrac{s_n^2 J_0^2(s_n r_o)}{J_0^2(s_n r_o) - J_0^2(s_n r_i)} \left[J_0(s_n r) Y_0(s_n r_i) - J_0(s_n r_i) Y_0(s_n r) \right]}{\sqrt{\left(k s_n^2\right)^2 + \omega^2}} \right\}$$

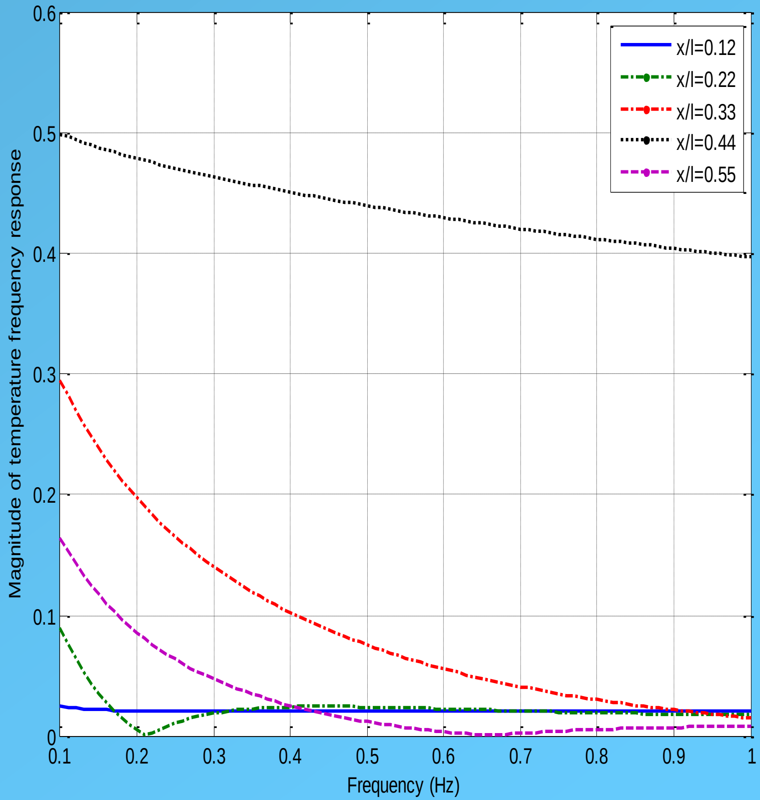
- Temperature fluctuations in the pipe-wall is given by

$$\Theta(r,\omega,t) \square \Theta_0 \left|H_T(r,\omega)\right| \times \sin\left[\omega t - \varphi\right]$$

Development of stochastic model for thermal FCG(4)
-Statistical properties of thermal spectrum



Comparison between predictions of temperature profile from complete analytical solution and those obtained by means of analytical temperature frequency response function in the pipe wall



Dependence of temperature frequency response magnitude on loading frequency for various depths through thickness (*l* is wall-thickness and *x* originates at inner pipe surface)

Development of stochastic model for thermal FCG(5)
-Statistical properties of thermal spectrum

- For one-dimensional equilibrium equation in the radial direction:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

- The hoop component is given in the following relationships in the case of plane strain [i]:

$$\sigma_\theta(r,\omega,t) = \frac{\alpha \cdot E}{1-\nu}\left[\frac{1}{r^2}\cdot I_1(r,\omega,t) + \frac{r^2 + r_i^2}{r^2\cdot(r_o^2 - r_i^2)}\cdot I_2(\omega,t) - \Theta(r,\omega,t)\right]$$

- The analytical form of integrals were developed in a previous paper [ii]; by using temperature FRF, they get the following forms:

$$I_1(r,\omega,t) \cong \Theta_0 \cdot \left[\sum_{n=1}^{\infty}\left|H_{I_1,n}(r,\omega,s_n)\right|\right]\cdot \sin(\omega t - \varphi)$$

$$I_2(\omega,t) \cong \Theta_0 \cdot \left[\sum_{n=1}^{\infty}\left|H_{I_2,n}(\omega,s_n)\right|\right]\cdot \sin(\omega t - \varphi)$$

[i] N. Noda, R.B. Hetnarski, Y. Tanigawa, Thermal Stresses, 2nd Ed., Taylor & Francis, 2003

[ii] V. Radu, E. Paffumi, N. Taylor, New analytical stress formulae for arbitrary time dependent thermal loads in pipes, European Commission Report EUR 22802 DG JRC, June 2007, Petten, NL.

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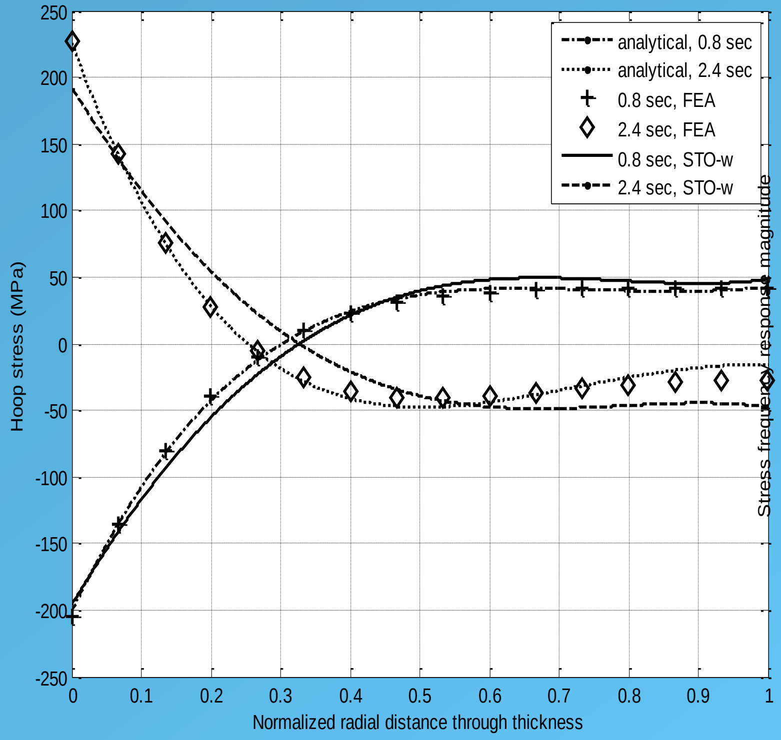
Development of stochastic model for thermal FCG(6) -Modeling of the stress response to stochastic thermal input

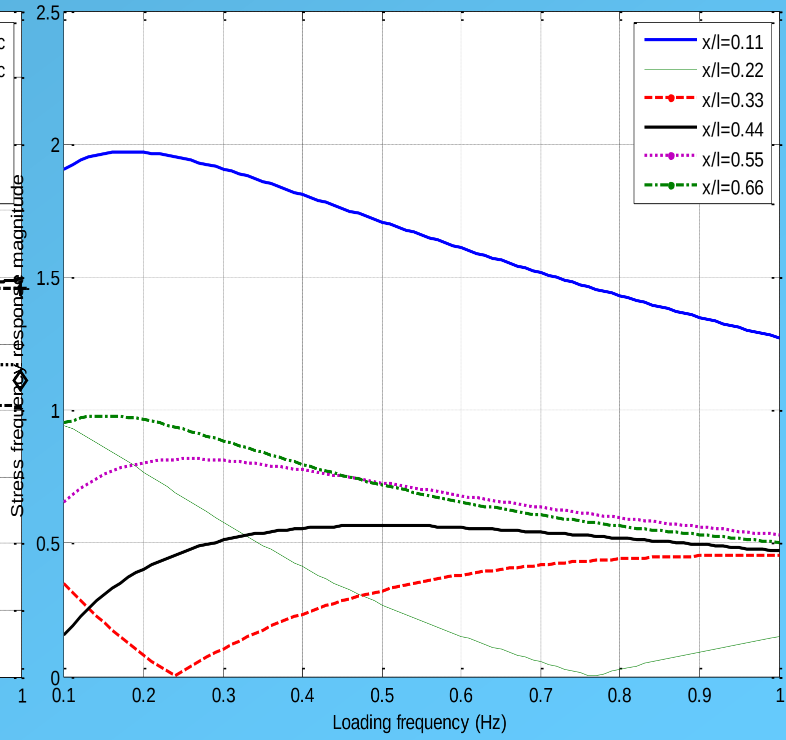
- As result, the hoop stress in the form of FRF is derived as:

$$\sigma_\theta(r,\omega,t) \approx \Theta_0 \cdot \left|H_{\sigma_\theta}(r,\omega)\right| \cdot \sin(\omega t - \varphi)$$

$$\begin{aligned}
\left|H_{\sigma_\theta}(r,\omega)\right| = abs\Bigg\{ & \frac{k\cdot\pi\cdot\alpha\cdot E}{1-\nu} \times \Bigg\{ \left(\frac{1}{r^2}\right) \sum_{n=1}^{\infty} \frac{s_n^2 \cdot J_0^2(s_n\cdot r_o)}{J_0^2(s_n\cdot r_o) - J_0^2(s_n\cdot r_i)} \times \\
& \times \left[\frac{Y_o(s_n\cdot r_i)\cdot[r\cdot J_1(s_n\cdot r) - r_i\cdot J_1(s_n\cdot r_i)] - J_o(s_n\cdot r_i)\cdot[r\cdot Y_1(s_n\cdot r) - r_i\cdot Y_1(s_n\cdot r_i)]}{s_n\sqrt{\left(ks_n^2\right)^2+\omega^2}} \right] \\
& + \frac{r^2+r_i^2}{r^2(r_o^2-r_i^2)} \times \sum_{n=1}^{\infty} \frac{s_n^2\cdot J_0^2(s_n\cdot r_o)}{J_0^2(s_n\cdot r_o) - J_0^2(s_n\cdot r_i)} \times \\
& \times \left[\frac{Y_o(s_n\cdot r_i)\cdot[r_o\cdot J_1(s_n\cdot r_o) - r_i\cdot J_1(s_n\cdot r_i)] - J_o(s_n\cdot r_i)\cdot[r_o\cdot Y_1(s_n\cdot r_o) - r_i\cdot Y_1(s_n\cdot r_i)]}{s_n\sqrt{\left(ks_n^2\right)^2+\omega^2}} \right] - \\
& - \sum_{n=1}^{\infty} \frac{s_n^2\cdot J_0^2(s_n\cdot r_o)}{J_0^2(s_n\cdot r_o) - J_0^2(s_n\cdot r_i)} \cdot \left[\frac{Y_o(s_n\cdot r_i)\cdot J_o(s_n\cdot r) - J_o(s_n\cdot r_i)\cdot Y_o(s_n\cdot r)}{s_n\sqrt{\left(ks_n^2\right)^2+\omega^2}} \right] \Bigg\} \Bigg\}
\end{aligned}$$

Development of stochastic model for thermal FCG(7)
-Modeling of the stress response to stochastic thermal input



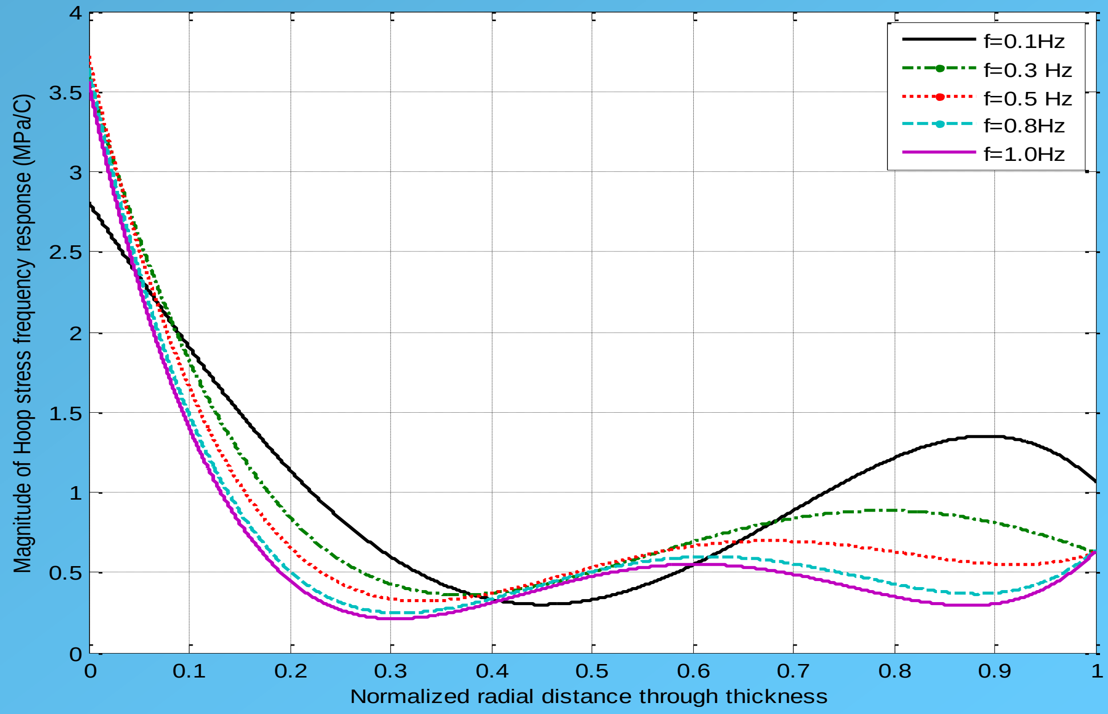


Comparison between predictions for hoop stress: complete analytical solution, FEA, and by means of stress frequency response function (f=0.3 Hz)

Magnitude of stress frequency response function versus loading frequency inside of the pipe-wall

Development of stochastic model for thermal FCG(8) -Modeling of the stress response to stochastic thermal input

- The dependence of profiles through thickness for stress frequency response magnitude, on the loading frequency



Magnitude of stress frequency response function through wall thickness versus loading frequency inside of the pipe-wall

Development of stochastic model for thermal FCG(9) - The SIF frequency response function

- The general form of the fourth order polynomial distribution is [i]:

$$\sigma(x) = \sigma_0 + \sigma_1 \cdot \left(\frac{x}{l}\right) + \sigma_2 \cdot \left(\frac{x}{l}\right)^2 + \sigma_3 \cdot \left(\frac{x}{l}\right)^3 + \sigma_4 \cdot \left(\frac{x}{l}\right)^4$$

- The SIF, *KI*, for surface crack under thermal stresses

$$K_I\left(\frac{a}{l}\right) = \sqrt{\frac{\pi a}{Q}} \cdot \left[G_0\sigma_0 + G_1\sigma_1 \cdot \left(\frac{a}{l}\right) + G_2\sigma_2 \cdot \left(\frac{a}{l}\right)^2 + G_3\sigma_3 \cdot \left(\frac{a}{l}\right)^3 + G_4\sigma_4 \cdot \left(\frac{a}{l}\right)^4 \right]$$

[i] API 579 Fitness-for-Service-API Recommended Practice 579, First Edition, January 2000, American Petroleum Institute.

Development of stochastic model for thermal FCG(10) - The SIF frequency response function

- The magnitude of frequency transfer function for SIF may be written in terms of the stress frequency response function [i]

$$\left|H_{\sigma_\theta}\left(\frac{x}{l},\omega\right)\right| = h_0(\omega) + h_1(\omega)\cdot\left(\frac{x}{l}\right) + h_2(\omega)\cdot\left(\frac{x}{l}\right)^2 + h_3(\omega)\cdot\left(\frac{x}{l}\right)^3 + h_4(\omega)\cdot\left(\frac{x}{l}\right)^4$$

$$\left|H_K\left(\frac{a}{l},\omega\right)\right| = \sqrt{\pi a}\cdot\left|G_K\left(\frac{a}{l},\omega\right)\right|$$

$$\left|G_K\left(\frac{a}{l},\omega\right)\right| = abs\left\{ h_0(\omega)\cdot G_0\left(\frac{a}{l}\right) + h_1(\omega)\cdot G_1\left(\frac{a}{l}\right)\cdot\left(\frac{a}{l}\right) + h_2(\omega)\cdot G_2\left(\frac{a}{l}\right)\cdot\left(\frac{a}{l}\right)^2 + \right.$$

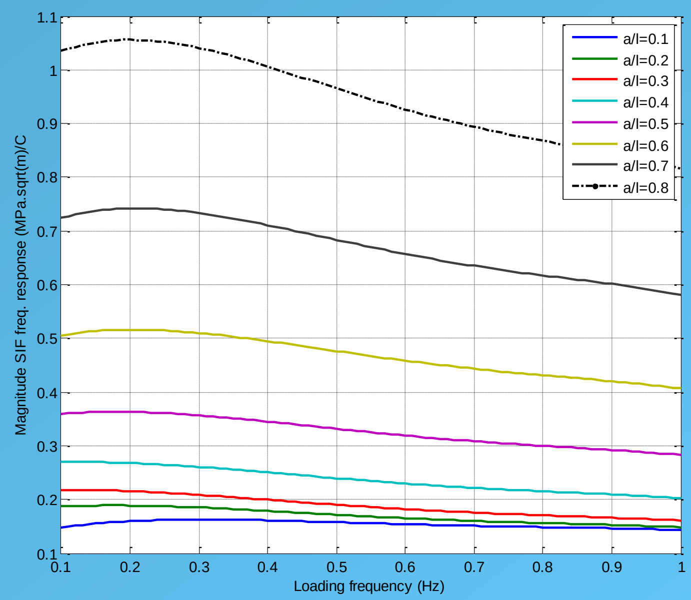
$$\left. + h_3(\omega)\cdot G_3\left(\frac{a}{l}\right)\cdot\left(\frac{a}{l}\right)^3 + h_4(\omega)\cdot G_4\left(\frac{a}{l}\right)\cdot\left(\frac{a}{l}\right)^4 \right\}$$

$$K\left(\frac{a}{l},\omega,t\right) = \Theta_0\cdot\sqrt{\pi a}\cdot\left|G_K\left(\frac{a}{l},\omega\right)\right|\cdot\sin(\omega t - \varphi)$$

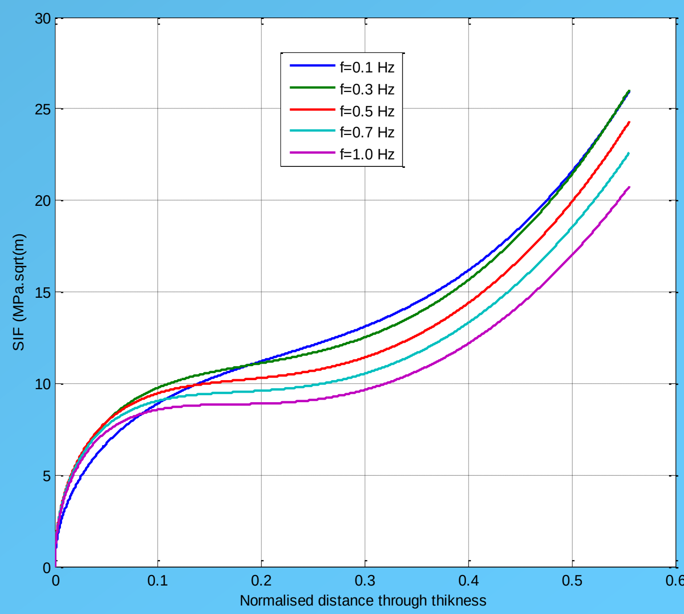
- [i]I.S. Jones, M.W. Lewis,A frequency response method for calculating stress intensity factors due to thermal striping loads, Fatigue and Fracture of Eng. Materials and Structures, vol.17, no.6, pp.709-720, 1994

Development of stochastic model for thermal FCG(11) - Derive the SIF frequency response function

The examination of this behavior of *KI*, which is calculated for instant of time $t=T/4$ (with T= time period of loading), suggests the highest its value for frequency f=0.3 *Hz*, which is in a good agreement with previous study [i]



Magnitude of SIF frequency response as function on loading frequency (*Hz*) on crack depth



Stress intensity factor (instant T/4) using SIF frequency response function

[i] V. Radu, E. Paffumi, N. Taylor, K.-F. Nilsson, A study on fatigue crack growth in high cycle domain assuming sinusoidal thermal loading, International Journal of Pressure Vessels and Piping (2009), doi:10.1016/j.ijpvp.2009.10.007].

Development of stochastic model for thermal FCG(12) - PSD of SIF and its spectral moments

- The approach followed here is to consider the temperature fluctuation and its spectrum as a Gaussian stationary narrow-band process

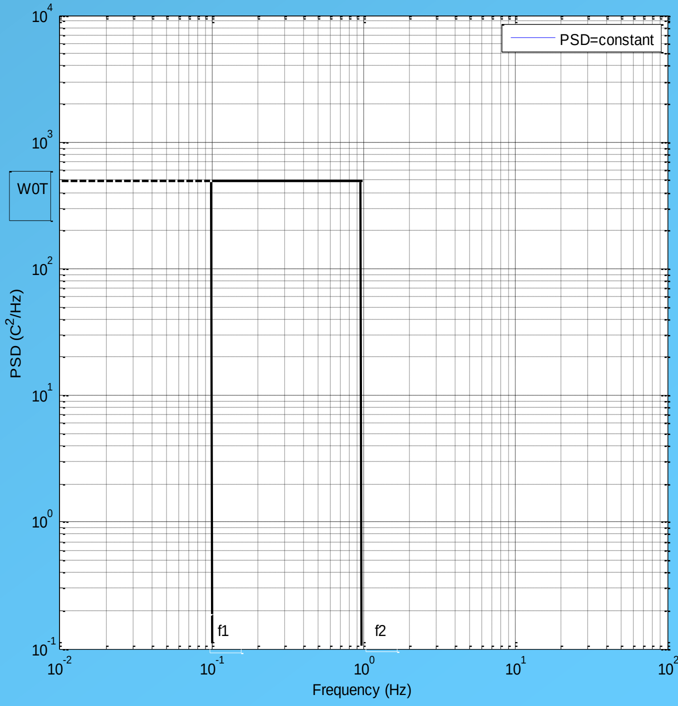
$$S_K(x_a,\omega)=\left|H_K(x_a,\omega)\right|^2 S_T(\omega)$$

$$R_T(\tau=0)=\sigma_T^2=\int_{f_1}^{f_2} W_T(f)df = W_{0T}(f_2-f_1)=W_{0T}\Delta f$$

$$W_K(x_a,f)=\left|H_K(x_a,f)\right|^2 W_T(f)$$

$$W_T(f)=\begin{cases} W_{0T}=ct, f\in[f_1,f_2] \\ 0, f\in[0,f_1)\cup(f_2,\infty] \end{cases}$$

$$W_K(x_a,f)=\left|H_K(x_a,f)\right|^2 W_{0T}$$



One-sided PSD for temperature fluctuations

Development of stochastic model for thermal FCG(13) - PSD of SIF and its spectral moments

- The zero order moment of *KI* PSD, which means SIF's variance for a normalized stationary Gaussian stochastic process, is given by

$$K_{rms}^{2}\left(x_a,\tau=0\right)=\int_{f_1}^{f_2} W_K\left(x_a,f\right)=\int_{f_1}^{f_2}\left|H_K\left(x_a,f\right)\right|^2 W_{0T}df=W_{0T}\int_{f_1}^{f_2}\left|H_K\left(x_a,f\right)\right|^2 df$$

- the expected rate of zero up crossing rates of *KI* is

$$\dot{N}_{K,0}=\dot{N}_{K,p}=\sqrt{\frac{\int_{f_1}^{f_2} f^2 W_K\left(x_a,f\right)df}{\int_{f_1}^{f_2} W_K\left(x_a,f\right)df}}=\sqrt{\frac{\int_{f_1}^{f_2} f^2\left|H_K\left(x_a,f\right)\right|^2 df}{\int_{f_1}^{f_2}\left|H_K\left(x_a,f\right)\right|^2 df}}$$

- The frequency of peaks of any magnitude for *KI*, stationary narrow band Gaussian process, is Rayleigh distribution

$$f_{K_I}\left(x_a,\tau=0\right)=\frac{K_I}{K_{rms}^2\left(x_a,\tau=0\right)}\exp\left[-\frac{1}{2}\frac{K_I}{K_{rms}^2\left(x_a,\tau=0\right)}\right]$$

Development of stochastic model for thermal FCG(14) - Expected value of thermal fatigue crack growth

- when time-dependent stochastic analysis is conducted, the crack growth rate of a random flaw size, *a*, should be written in the following form:

$$\frac{da}{dt} = \frac{da}{dN}\frac{dN}{dt} = \nu_p \frac{da}{dN}$$

  - where *vp* is the mean rate of maxima, that is constant for a Gaussian stationary stochastic process.

$$\nu_p = \dot{N}_{K,0} = \dot{N}_{K,p} = \sqrt{\frac{\int_{f_1}^{f_2} f^2 W_K(x_a, f)\,df}{\int_{f_1}^{f_2} W_K(x_a, f)\,df}} = \sqrt{\frac{\int_{f_1}^{f_2} f^2 \left|H_K(x_a, f)\right|^2 df}{\int_{f_1}^{f_2} \left|H_K(x_a, f)\right|^2 df}}$$

- We assume a linear summation of damage and ignore the effect of positive minima [i] in this case the expectation rate of crack growth in respect to cycles is

$$E\left[\frac{da}{dN}\right] = C \cdot \int_0^{\infty} (K_I)^n \frac{K_I}{K_{rms}^2(x_a, \tau = 0)} \exp\left[-\frac{1}{2}\frac{K_I}{K_{rms}^2(x_a, \tau = 0)}\right] dK_I$$

[i] A.G. Miller, Crack propagation due to random thermal fluctuation: effect of temporal incoherence, International Journal of Pressure vessels and Piping, 8 (1980), pp.15-24

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Development of stochastic model for thermal FCG(15) - Lifetime assessment for thermal fatigue crack growth

- The final form of stochastic crack growth rate is

$$\frac{dx_a}{dt} = \frac{C}{l} \cdot \sqrt{\frac{\int_{f_1}^{f_2} f^2 \left|H_K(x_a, f)\right|^2 df}{\int_{f_1}^{f_2} \left|H_K(x_a, f)\right|^2 df}} \left[W_{0T} \int_{f_1}^{f_2} \left|H_K(x_a, f)\right|^2 df \right]^n \cdot 2^{\frac{n}{2}} \Gamma\left(1 + \frac{1}{n}\right)$$

- To combine the stochastic behavior of *K* with statistical characteristics of crack growth under constant amplitude (*C* and *n* Paris law parameters), and also with initial crack depth distribution, we define the limit state function in the form

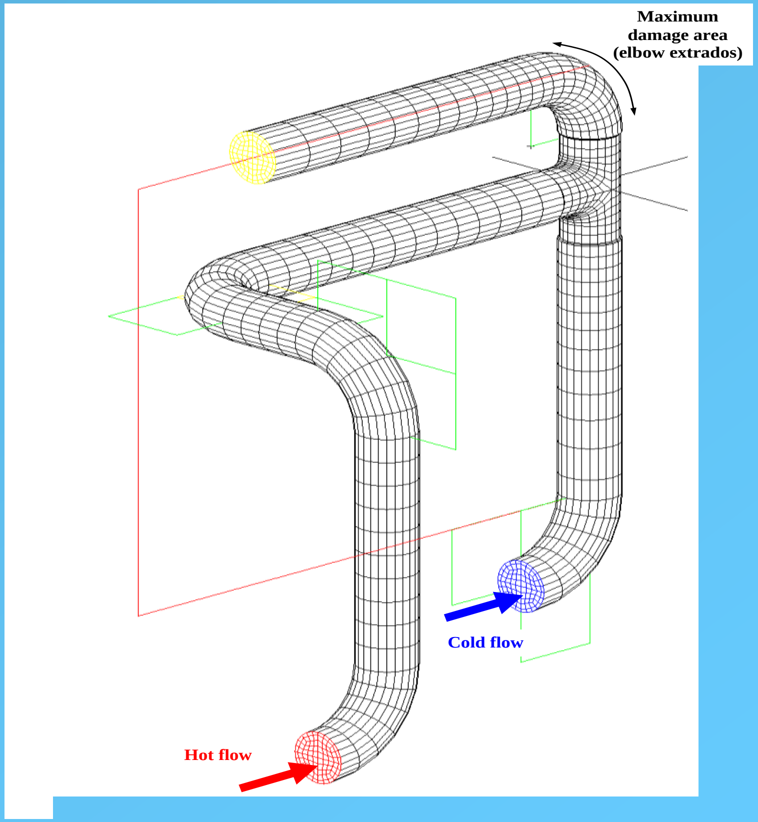
$$g(t_{ref}) = 1 - \frac{t_{stoch}}{t_{ref}}$$

- During the Monte Carlo simulation (MCS), the trials which satisfy condition

$$g\left(t_{ref}\right) \le 0 \qquad\qquad P_f = \frac{n_{fail}}{N_{trials}}$$

Application: Civaux 1 damage case (1)

- In 1998 a longitudinal crack was discovered at outer edge of an elbow in a mixing zone of the RHRS of the Civaux NPP unit 1
  - the origin cracking by thermal fatigue.
  - the time between initiation of the crack and its development to a significant depth through the wall was only about ≈1500 hours
  - pressure of 36 bar, the hot leg contains water at 180 C and the cold leg contains water at 20 C.
  - in the damage zone of interest the pipe *ri* ≅120 mm and *ro* =129 mm.
- The temperature fluctuation was reported to be in the range 20-180C and on the inner surface of the pipe the maximum temperature fluctuation range was estimated to be 120C



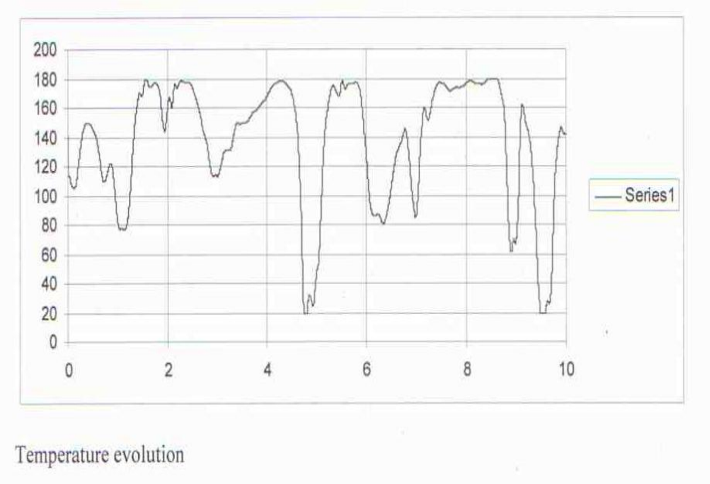
The simplified sketch of piping subsystems with damaged area by thermal fatigue cracking [i].

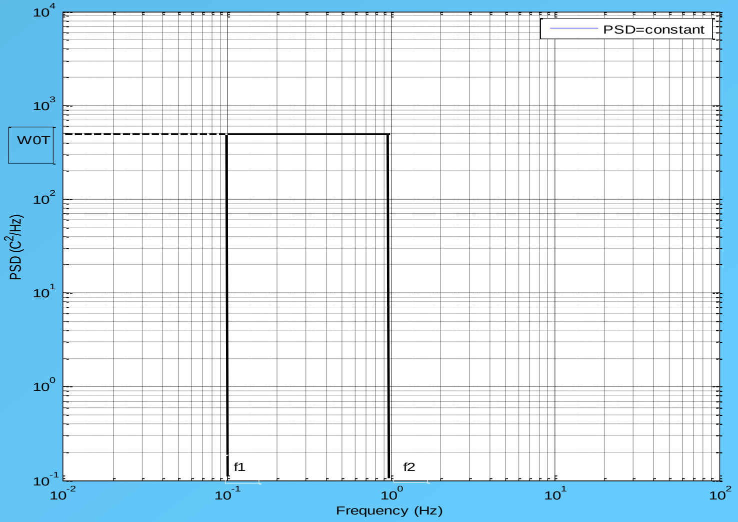
[[i]] S. Chapuliot, C. Gourdin, T. Payen, J.P. Magnaud, A. Monavon,Hydro-thermal-mechanical analysis of thermal fatigue in a mixing tee, Nuclear engineering and Design 235 (2005) 575-596

Application: Civaux 1 damage case (2)

- For a thermal spectrum assumed to be stationary Gaussian stochastic process we use the one-sided temperature PSD:

$$W_T(f) = \begin{cases} W_{0T} = 500C^2/Hz,\ f \in [0.1, 1.0] \quad Hz \\ 0,\ f \in [0, 0.1Hz) \cup (1.0Hz, \infty] \end{cases}$$



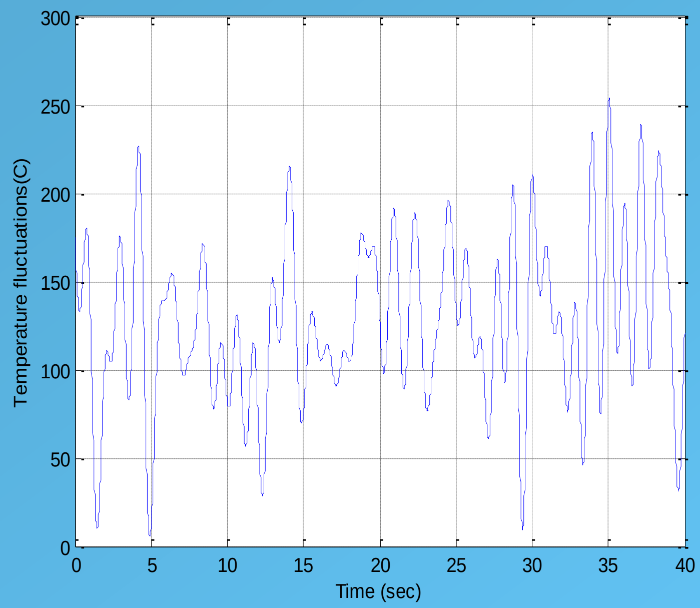


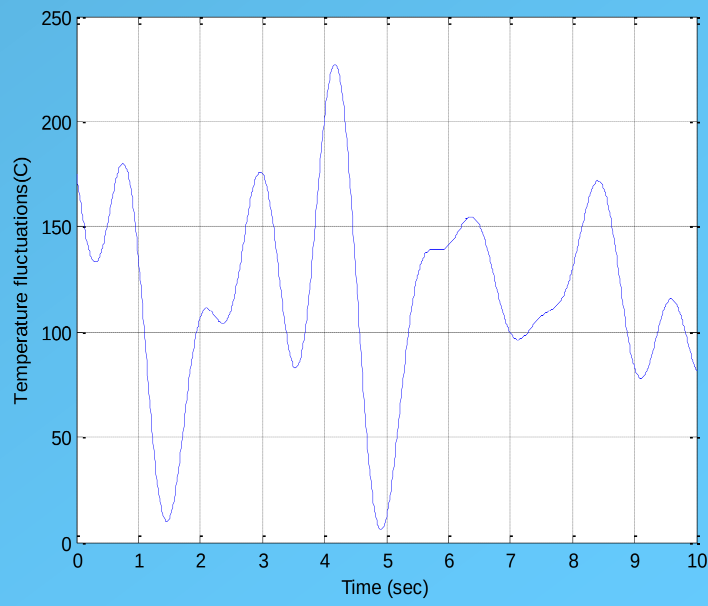
Temperature fluctuations Ciavux 1 case [i]

Model of the one-sided PSD of temperature fluctuations - Civaux 1 case

C. Gourdin, S. Marie, S. Chapuliot, An Analytical Thermal Fatigue crack growth approach, SMiRT 20-Division 2, Paper 1796, Espoo, Finland, August 9-14, 2009

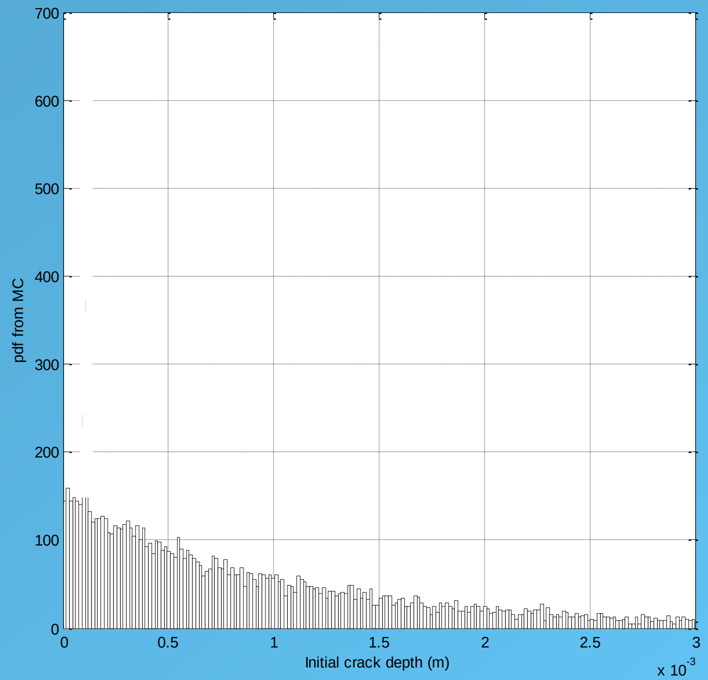
Application: Civaux 1 damage case (3)



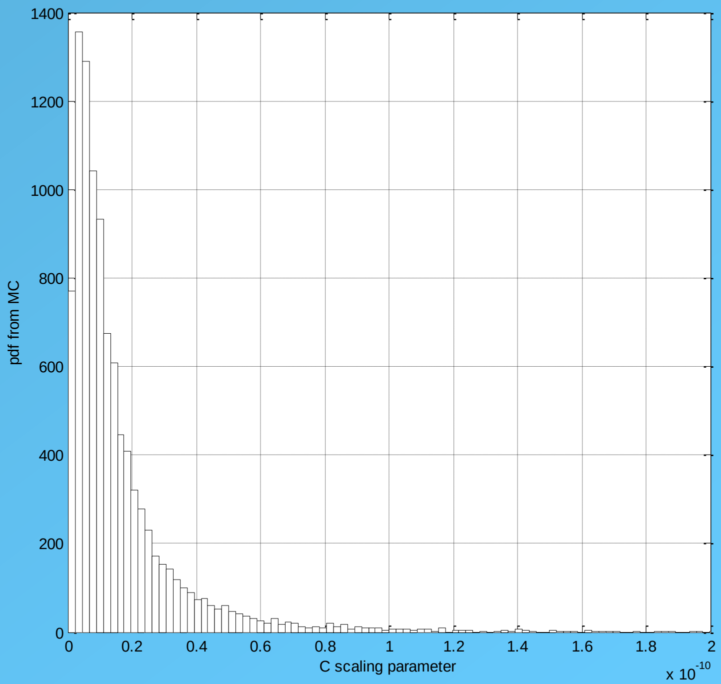


Sample function (RSA) of the temperature variation using the one-sided model of PSD of a stationary Gaussian narrow-band process, (W0T=500 C^2/Hz, frequency range=0.1-1.0 Hz, with non-zero mean value

Application: Civaux 1 damage case (4)



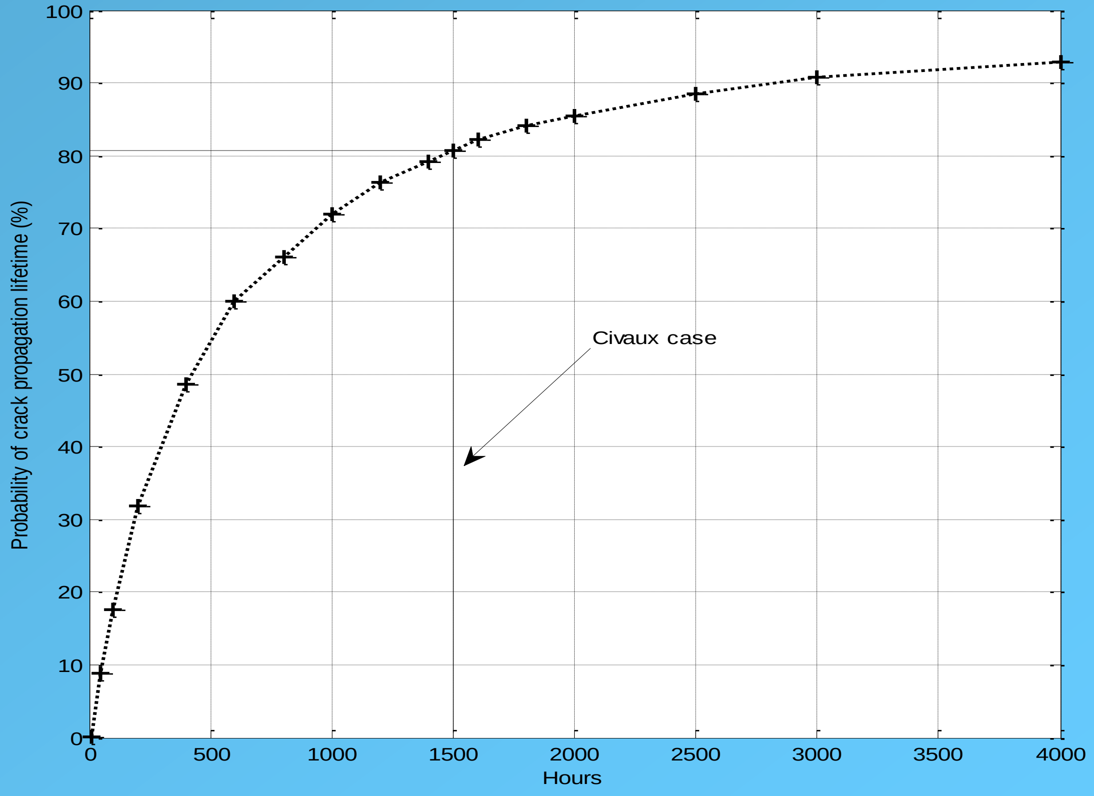
Probability density function for initial crack depth (MC simulations)



Probability density function for *C* scaling parameter (MC simulations)

Application: Civaux 1 damage case (5)

- Probabilities of failure: the stochastic modeling results of fatigue crack growth coupled with probabilistic input in Paris law for Monte Carlo simulation.



Conclusions and further developments

- The study proposes a stochastic model to assess thermal fatigue crack growth in mixing tees of NPP with the temperature spectrum assumed to be a Gaussian stationary narrow-band stochastic process. The stochastic fatigue crack growth model includes a main part for incorporating randomness in service loads, and also another one which includes a description of statistical characteristics of crack growth under constant amplitude loadings.
- Based on the analytical solution of temperature response (Hankel transform) within SIN-methodology developed in previous work, a temperature frequency response function through pipe thickness is developed.
- By considering the analytical solution for thermal stresses also developed in previous works, a stress frequency response function for thermal hoop stress is derived and based on this a SIF frequency response magnitude is obtained.
- For a one-sided PSD model of temperature fluctuation, the PSD of SIF is obtained, by means of FRF methodology and, consequently, the expected value of crack growth rate in HCF domain can be assessed using the Rayleigh distribution moments.
- The variability in Paris law parameter is accounted and in crack distribution as well, and the probabilities of failure are obtained by MCS (Civaux case).
- The present methodology based on the stochastic modeling of thermal fatigue crack growth can be used to analyze and improve the screening criteria proposed to avoid cracking issues in nuclear piping, especially in tee connections where turbulent mixing of flows with different temperature can occur.
- Further developments:
  - application to gen. IV issues ;
  - various PSD forms, more complex,
  - coupling with stochastic crack initiation (stochastic- spatial incoherence);
  - different Paris law with non dimensional ΔK to avoid strong corelation between C and n;
  - rain flow method coupling;
  - the influence of stress-free temperature.

- Thank you

for your attention!